[This question paper contains 3 printed pages.]

Sr. No. of Question Paper : 8895 (12) **Your Roll No.** 2019

Unique Paper Code : 235101

Name of the Course : **B.Sc. (Hons.) Mathematics**

Name of the Paper : MAHT 101 Calculus

Semester : I

Duration: 3 Hours Maximum Marks: 75

Instruction for Candidates

1) Write your Roll No. on the top immediately on receipt of this question paper.
2) All the sections are compulsory.
3) All questions carry equal marks.
4) Use of non-programmable Scientific Calculators is allowed.

Section I

Attempt any **four** *questions from* **Section I**.

1. If $y = (1 - x^2)^{-\frac{1}{2}} Sin^{-1}x$, when $-1 < x < 1$ and $-\frac{\pi}{2} < Sin^{-1}x < \frac{\pi}{2}$, then show that $(1 - x^2)y_{n+1} - (2n + 1)xy_n - n^2y_{n-1} = 0$

2. Sketch the graph of the function $f(x) = 4 + \frac{2x}{x-3}$ by determining all critical points, interval of increase and decrease, point of relative maxima and minima, concavity of the graph, inflection point and horizontal and vertical asymptotes.

3. Evaluate :

$$\lim_{x \to +\infty} [x - \log(x^3 - 1)]$$

4. Sketch the graph of $r = 5 - 2Cos\theta$ in polar coordinates.

5. When the market price of a certain product is p, then number of units sold will be

$$x = -6 \log\left(\frac{P}{40}\right)$$

It is also estimated that the cost of producing these x units will be

$$C(x) = 4xe^{\left(-\frac{x}{6}\right)} + 30$$

(a) Find the average cost, the marginal cost, and the marginal revenue for this production process.

(b) What level of production x corresponds to maximum profit?

Section II

Attempt any **four** *questions from* **Section II**.

6. Find the reduction formula for $\int \sec^n x\, dx$ where $n \geq 2$ is an integer. Hence, evaluate $\int \sec^5 x\, dx$.

7. Find the volume of solid that results when the region enclosed by $x = y^2$ and $x = y$ is revolved about the line $y = -1$.

8. Use cylindrical shell method to find the volume of the solid generated when the region enclosed by the curves $xy = 6, x + y = 5$ is revolved about the $x-$axis.

9. Find the arc length of the parametric curve $x = a(t + Sin\, t)$, $y = a(1 - \cos t)$ for $-\pi \leq t \leq \pi$.

10. Find the area of the surface generated by revolving the parametric curve $x = Cos^2 t, y = Sin^2 t, 0 \leq t \leq \pi/2$ about the y-axis.

Section III

Attempt any **four** *questions from* **Section III**.

11. Find the equation of the hyperbola passing through the origin with asymptotes $y = 2x + 1$ and $y = -2x + 3$.

12. Find the equation of the ellipse having foci at $(0, \pm 6)$, length of the minor axis 16

13. Identify and sketch the following curve :

$$153x^2 - 192xy + 97y^2 - 30x - 40y - 200 = 0.$$

14. Identify and sketch the following curve :

$$y^2 - 8x - 6y - 23 = 0.$$

Section IV

Attempt any **four** *questions from* **Section IV**.

15. If F(t) is a differentiable vector valued function of t of constant length then show that F(t) is orthogonal to its derivative for all t.

16. Evaluate $\int_0^{\frac{\pi}{4}} F(t)\, dt$, where $F(t) = (\sec^2 t, -2\cos t, 1)$.

17. Express the acceleration of the particle in the form $a_T T + a_N N$, where T is the unit tangent vector and N is the unit normal vector, given that the particle moves so that its position at any time t is $r(t) = (e^t \cos t, e^t \sin t, \sqrt{2}e^t)$, $t > 0$.

18. Find the curvature and radius of curvature of the twisted cubic for a curve $\mathbf{r}(t) = (t, t^2, t^3)$ at a general point and at $(0, 0, 0)$.

19. A projectile is fired from ground level at angle 30° with muzzle speed of 80 ft/s. Find time of flight and the range.