[This question paper contains 3 printed pages.]

Sr. No. of Question Paper :

Your Roll No. 2019

Unique Paper Code

: 235101

Name of the Course

: B.Sc. (Hons.) Mathematics

Name of the Paper

: MAHT 101 Calculus

Semester

: 1

Duration: 3 Hours

Maximum Marks: 75

Instruction for Candidates

- 1) Write your Roll No. on the top immediately on receipt of this question paper.
- All the sections are compulsory.
- 3) All questions carry equal marks.
- 4) Use of non-programmable Scientific Calculators is allowed.



Section I

Attempt any four questions from Section I.

- If $y = (1 x^2)^{-\frac{1}{2}} Sin^{-1}x$, when -1 < x < 1 and $-\frac{\pi}{2} < Sin^{-1}x < \frac{\pi}{2}$, then show that $(1 x^2)y_{n+1} (2n + 3)xy_n n^2y_{n-1} = 0$ 1.
- Sketch the graph of the function $f(x) = 4 + \frac{2x}{x-3}$ by determining all critical points, interval 2. of increase and decrease, point of relative maxima and minima, concavity of the graph, inflection point and horizontal and vertical asymptotes.
- 3. Evaluate:

$$\lim_{x \to +\infty} [x - \log(x^3 - 1)]$$

- Sketch the graph of $r = 5 2Cos\theta$ in polar coordinates. 4.
- When the market price of a certain product is p, then number of units sold will be 5.

$$x = -6\log\left(\frac{P}{40}\right)$$

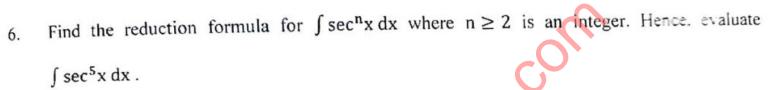
It is also estimated that the cost of producing these x units will be

$$C(x) = 4xe^{\left(-\frac{x}{6}\right)} + 30$$

- (a) Find the average cost, the marginal cost, and the marginal revenue for this production process.
- (b) What level of production x corresponds to maximum profit?

Section II

Attempt any four questions from Section II.



7. Find the volume of solid that results when the region enclosed by $x = y^2$ and x = y is revolved about the line y = -1.

8. Use cylindrical shell method to find the volume of the solid generated when the region enclosed by the curves xy = bx + y = 5 is revolved about the x - axis.

9. Find the arc length of the parametric curve x = a(t + Sin t), $y = a(1 - \cos t)$ for $-\pi \le t \le \pi$.

10. Find the area of the surface generated by revolving the parametric curve $x = Cos^2t$, $y = Sin^2t$, $0 \le t \le \frac{\pi}{2}$ about the y-axis.

Section III

Attempt any four questions from Section III.

11. Find the equation of the hyperbola passing through the origin with asymptotes y = 2x + 1 and y = -2x + 3.

- 12. Find the equation of the ellipse having foci at (0, ±6), length of the minor axis 16
- 13. Identify and sketch the following curve:

$$153x^2 - 192xy + 97y^2 - 30x - 40y - 200 = 0.$$

14. Identify and sketch the following curve:

$$y^2 - 8x - 6y - 23 = 0.$$



Section IV

Attempt any four questions from Section IV.

- 15. If F(t) is a differentiable vector valued function of t of constant length then show that F(t) is orthogonal to its derivative for all t.
- 16. Evaluate $\int_0^{\frac{\pi}{4}} F(t) dt$, where $F(t) = (\sec^2 t, -2 \cos t, 1)$.
- 17. Express the acceleration of the particle in the form $a_T T + a_N N$, where T is the unit tangent vector and N is the unit normal vector, given that the particle moves so that its position at any time t is $r(t) = (e^t \cos t, e^t \sin t, \sqrt{2}e^t)$, t > 0.
- 18. Find the curvature and radius of curvature of the twisted cubic for a curve $\mathbf{r}(t) = (t, t^2, t^3)$ at a general point and at (0,0,0).
- 19. A projectile is fired from ground level at angle 30° with muzzle speed of 80 ft/s. Find time of flight and the range.